UR ACADEMY, BANGALORE

KCET 2023-24 Mathematics – Version D4

1. The value of C in (0, 2) satisfying the mean value theorem for the function $f(x) = x\,(x-1)^2,\ x \in [0, 2]$ is equal to

   (A) $\frac{3}{4}$ (B) $\frac{4}{3}$ (C) $\frac{1}{3}$ (D) $\frac{2}{3}$

   Ans: (B)

2. $\frac{d}{dx}\left[\cos^2\left(\cot^{-1}\sqrt{\frac{2+x}{2-x}}\right)\right]$ is

   (A) $-\frac{3}{4}$ (B) $-\frac{1}{2}$

   (C) $\frac{1}{2}$ (D) $\frac{1}{4}$

   Ans: (D)

3. For the function $f(x) = x^3 - 6x^2 + 12x - 3$; $x = 2$ is

   (A) a point of minimum (B) a point of inflexion

   (C) not a critical point (D) a point of maximum

   Ans: (B)

4. The function $f(x) = |\cos x|$ is

   (A) everywhere continuous and differentiable

   (B) everywhere continuous but not differentiable at odd multiples of $\frac{\pi}{2}$

   (C) neither continuous nor differentiable at $(2n+1)\frac{\pi}{2}, n \in Z$

   (D) not differentiable everywhere

   Ans: (B)

5. If $y = 2x^{3x}$, then $\frac{dy}{dx}$ at $x = 1$ is

   (A) 2 (B) 6 (C) 3 (D) 1

   Ans: (B)

6. Let the function satisfy the equation $f(x + y) = f(x)\,f(y)$ for all $x, y \in R$, where $f(0) \neq 0$. If $f(5) = 3$ and $f'(0) = 2$, then $f'(5)$ is

   (A) 6 (B) 0

   (C) 5 (D) – 6

   Ans: (A)

7. $\int \frac{1}{x\left[6(\log x)^2 + 7\log x + 2\right]} dx =$

(A) $\frac{1}{2}\log\left|\frac{2\log x+1}{3\log x+2}\right| + C$

(B) $\log\left|\frac{2\log x+1}{3\log x+2}\right| + C$

(C) $\log\left|\frac{3\log x+2}{2\log x+1}\right| + C$

(D) $\frac{1}{2}\log\left|\frac{3\log x+2}{2\log x+1}\right| + C$

Ans: (B)

8. $\int \frac{\sin\frac{5x}{2}}{\sin\frac{x}{2}} dx =$

(A) 2x + sin x + 2 sin 2x + C

(B) x + 2 sin x + 2 sin 2x + C

(C) x + 2 sinx + sin 2x + C

(D) 2x + sin x + sin 2x + C

Ans: (C)

9. $\int_1^5 \left(|x-3| + |1-x|\right) dx =$

(A) 12 (B) $\frac{5}{6}$ (C) 21 (D) 10

Ans: (A)

10. $\lim_{n\to\infty}\left(\frac{n}{n^2+1^2} + \frac{n}{n^2+2^2} + \frac{n}{n^2+3^2} + ... + \frac{1}{5n}\right) =$

(A) $\frac{\pi}{4}$ (B) $\tan^{-1}3$ (C) $\tan^{-1}2$ (D) $\frac{\pi}{2}$

Ans: (C)

11. The area of the region bounded by the line y = 3x and the curve $y = x^2$ in sq. units is

(A) 10 (B) $\frac{9}{2}$ (C) 9 (D) 5

Ans: (B)

12. The area of the region bounded by the line y = x and the curve $y = x^3$ is

(A) 0.2 sq. units

(B) 0.3 sq. units

(C) 0.4 sq. units

(D) 0.5 sq. units

Ans: (D)

13. If $\vec{a}, \vec{b}, \vec{c}$ are three non-coplanar vectors and p, q, r are vectors defined by

$\vec{p} = \frac{\vec{b}\times\vec{c}}{\left[\vec{a}\ \vec{b}\ \vec{c}\right]}, \vec{q} = \frac{\vec{c}\times\vec{a}}{\left[\vec{a}\ \vec{b}\ \vec{c}\right]}, \vec{r} = \frac{\vec{a}\times\vec{b}}{\left[\vec{a}\ \vec{b}\ \vec{c}\right]}$, then

$\left(\vec{a}+\vec{b}\right).\vec{p} + \left(\vec{b}+\vec{c}\right).\vec{q} + \left(\vec{c}+\vec{a}\right).\vec{r}$ is

(A) 0 (B) 1 (C) 2 (D) 3

Ans: (D)

14. If lines $\frac{x-1}{-3}=\frac{y-2}{2k}=\frac{z-3}{2}$ and $\frac{x-1}{3k}=\frac{y-5}{1}=\frac{z-6}{-5}$ are mutually perpendicular, then k is equal to

(A) $-\frac{10}{7}$ (B) $-\frac{7}{10}$ (C) – 10 (D) – 7

Ans: (A)

15. The distance between the two planes 2x + 3y + 4z = 4 and 4x + 6y + 8z = 12 is

(A) 2 units (B) 8 units (C) $\frac{2}{\sqrt{29}}$ units (D) 4 units

Ans: (C)

16. The sine of the angle between the straight line $\frac{x-2}{3}=\frac{y-3}{4}=\frac{4-z}{-5}$ and the plane $2x-2y+z=5$ is

(A) $\frac{1}{5\sqrt{2}}$ (B) $\frac{2}{5\sqrt{2}}$ (C) $\frac{3}{50}$ (D) $\frac{3}{\sqrt{50}}$

Ans: (A)

17. The equation xy = 0 in three-dimensional space represents
(A) a pair of straight lines
(B) a plane
(C) a pair of planes at right angles
(D) a pair of parallel planes
Ans: (C)

18. The plane containing the point (3, 2, 0) and the line $\frac{x-3}{1}=\frac{y-6}{5}=\frac{z-4}{4}$ is

(A) $x-y+z=1$ (B) $x+y+z=5$
(C) $x+2y-z=1$ (D) $2x-y+z=5$

Ans: (A)

19. Corner points of the feasible region for un LPP are (0, 2), (3, 0), (6, 0), (6, 8) and (0, 5). Let z = 4x + 6y be the objective function. The minimum value of z occurs at
(A) Only (0, 2)
(B) Only (3, 0)
(C) The mid-point of the line segment joining the points (0, 2) and (3, 0)
(D) Any point on the line segment joining the points (0, 2) and (3, 0)
Ans: (D)

20. A die is thrown 10 times. The probability that an odd number will come up at least once is

(A) $\frac{11}{1024}$ (B) $\frac{1013}{1024}$
(C) $\frac{1023}{1024}$ (D) $\frac{1}{1024}$

Ans: (C)

21. A random variable X has the following probability distribution:

X	0	1	2
$P(X)$	$\frac{25}{36}$	k	$\frac{1}{36}$

If the mean of the random variable X is $\frac{1}{3}$, then the variance is

(A) $\frac{1}{18}$ (B) $\frac{5}{18}$ (C) $\frac{7}{18}$ (D) $\frac{11}{18}$

Ans: (B)

22. If a random variable X follows the binomial distribution with parameters n = 5, p and P(X = 2) = 9P(X = 3), then p is equal to

(A) 10 (B) $\frac{1}{10}$ (C) 5 (D) $\frac{1}{5}$

Ans: (B)

23. Two finite sets have m and n elements respectively. The total number of subsets of the first set is 56 more than the total number of subsets of the second set. The values of m and n respectively are

(A) 7, 6 (B) 5, 1 (C) 6, 3 (D) 8, 7

Ans: (C)

24. If $|x|^2 - 5|x| + 6 = 0$, where $|x|$ denotes the greatest integer function, then

(A) $x \in [3,4]$ (B) $x \in [2,4)$ (C) $x \in [2,3]$ (D) $x \in (2,3]$

Ans: (B)

25. If in two circles, arcs of the same length subtend angles 30° and 78° at the centre, then the ratio of their radii is

(A) $\frac{5}{13}$ (B) $\frac{13}{5}$ (C) $\frac{13}{4}$ (D) $\frac{4}{13}$

Ans: (B)

26. If Δ ABC is right angled at C, then the value of tan A + tan B is

(A) a + b (B) $\frac{a^2}{bc}$ (C) $\frac{c^2}{ab}$ (D) $\frac{b^2}{ac}$

Ans: (C)

27. The real value of 'α' for which $\frac{1 - i\sin\alpha}{1 + 2i\sin\alpha}$ is purely real is

(A) $(n+1)\frac{\pi}{2}, n \in \mathbb{N}$ (B) $(2n+1)\frac{\pi}{2}, n \in \mathbb{N}$ (C) $n\pi, n \in \mathbb{N}$ (D) $(2n-1)\frac{\pi}{2}, n \in \mathbb{N}$

Ans: (C)

28. The length of a rectangle is five times the breadth. If the minimum perimeter of the rectangle is 180 cm, then

(A) Breadth ≤ 15 cm (B) Breadth ≥ 15 cm

(C) Length ≤ 15 cm (D) Length = 15 cm

Ans: (B)

29. The value of ${}^{49}C_3 + {}^{48}C_3 + {}^{47}C_3 + {}^{46}C_3 + {}^{45}C_3 + {}^{45}C_4$ is
(A) ${}^{50}C_4$ (B) ${}^{50}C_3$ (C) ${}^{50}C_2$ (D) ${}^{50}C_1$
Ans: (A)

30. In the expansion of $(1+x)^n \ \frac{C_1}{C_0} + 2\frac{C_2}{C_1} + 3\frac{C_3}{C_2} + ... + n\frac{C_n}{C_{n-1}}$ is equal to
(A) $\frac{n(n+1)}{2}$ (B) $\frac{n}{2}$ (C) $\frac{n+1}{2}$ (D) $3n(n+1)$
Ans: (A)

31. If S_n stands for sum to n-terms of a G.P. with 'a' as the first term and 'r' as the common ratio then $S_n : S_{2n}$ is
(A) $r^n + 1$ (B) $\frac{1}{r^n + 1}$ (C) $r^n - 1$ (D) $\frac{1}{r^n - 1}$
Ans: (B)

32. If AM. and G.M. of roots of a quadratic equation are 5 and 4 respectively, then the quadratic equation is
(A) $x^2 - 10x - 16 = 0$ (B) $x^2 + 10x + 16 = 0$
(C) $x^2 + 10x - 16 = 0$ (D) $x^2 - 10x + 16 = 0$
Ans: (D)

33. The angle between the line x + y = 3 and the line joining the points (1, 1) and (– 3, 4) is
(A) $\tan^{-1}(7)$ (B) $\tan^{-1}\left(-\frac{1}{7}\right)$ (C) $\tan^{-1}\left(\frac{1}{7}\right)$ (D) $\tan^{-1}\left(\frac{2}{7}\right)$
Ans: (C)

34. The equation of parabola whose focus is (6, 0) and directrix is x = –6 is
(A) $y^2 = 24x$ (B) $y^2 = -24x$ (C) $x^2 = 24y$ (D) $x^2 = -24y$
Ans: (A)

35. $\lim_{x \to \frac{\pi}{4}} \frac{\sqrt{2}\cos x - 1}{\cot x - 1}$ is equal to
(A) 2 (B) $\sqrt{2}$ (C) $\frac{1}{2}$ (D) $\frac{1}{\sqrt{2}}$
Ans: (C)

36. The negation of the statement
"For every real number x; $x^2 + 5$ is positive" is
(A) For every real number x; $x^2 + 5$ is not positive.
(B) For every real number x; $x^2 + 5$ is negative.
(C) There exists at least one real number x such that $x^2 + 5$ is not positive.
(D) There exists at least one real number x such that $x^2 + 5$ is positive.
Ans: (C)

37. Let a, b, c, d and e be the observations with mean m and standard deviation S. The standard deviation of the observations a + k, b + k, c + k, d + k and e + k is

(A) kS (B) S + k (C) $\frac{S}{k}$ (D) S

Ans: (D)

38. Let $f: R \to R$ be given by $f(x) = \tan x$. Then $f^{-1}(1)$ is

(A) $\frac{\pi}{4}$ (B) $\left\{n\pi + \frac{\pi}{4} : n \in Z\right\}$ (C) $\frac{\pi}{3}$ (D) $\left\{n\pi + \frac{\pi}{3} : n \in Z\right\}$

Ans: (B)

39. Let $f: R \to R$ be defined by $f(x) = x^2 + 1$. Then the pre images of 17 and –3 respectively are

(A) $\phi, \{4, -4\}$ (B) $\{3, -3\}, \phi$ (C) $\{4, -4\}, \phi$ (D) $\{4, -4\}, \{2, -2\}$

Ans: (C)

40. Let $(gof)(x) = \sin x$ and $(fog)(x) = \left(\sin\sqrt{x}\right)^2$. Then

(A) $f(x) = \sin^2 x,\ g(x) = x$ (B) $f(x) = \sin\sqrt{x},\ g(x) = \sqrt{x}$

(C) $f(x) = \sin^2 x, g(x) = \sqrt{x}$ (D) $f(x) = \sin\sqrt{x}, g(x) = x^2$

Ans: (C)

41. Let A = {2, 3, 4, 5, …… 16, 17, 18}. Let R be the relation on the set A of ordered pairs of positive integers defined by (a, b) R (c, d) if and only ad = bc for all (a, b), (c, d) in $A \times A$. Then the number of ordered pairs of the equivalence class of (3, 2) is

(A) 4 (B) 5 (C) 6 (D) 7

Ans: (C)

42. If $\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = 3\pi$, then x (y + z) + y (z + x) + z (x + y) equals to

(A) 0 (B) 1 (C) 6 (D) 12

Ans: (C)

43. If $2\sin^{-1} x - 3\cos^{-1} x = 4, x \in [1,1]$ then $2\sin^{-1} x + 3\cos^{-1} x$ is equal to

(A) $\frac{4 - 6\pi}{5}$ (B) $\frac{\pi - 4}{5}$ (C) $\frac{3\pi}{2}$ (D) 0

Ans: (B)

44. If A is a square matrix such that $A^2 = A$, then $(I + A)^3$ is equal to

(A) 7A − I (B) 7A (C) 7A + I (D) I − 7A

Ans: (C)

45. If $A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$, then A^{10} is equal to

(A) 2^8 A (B) 2^9 A (C) 2^{10} A (D) 2^{11} A

Ans: (B)

46. If $f(x)=\begin{vmatrix} x-3 & 2x^2-18 & 2x^3-81 \\ x-5 & 2x^2-50 & 4x^3-500 \\ 1 & 2 & 3 \end{vmatrix}$, then $f(1)\cdot f(3)+f(3)\cdot f(5)+f(5)\cdot f(1)$ is

(A) −1 (B) 0 (C) 1 (D) 2

Ans: (GRACE)

47. Let $f(x)=\begin{vmatrix} \cos x & x & 1 \\ 2\sin x & x & 2x \\ \sin x & x & x \end{vmatrix}$. Then $\lim_{x\to 0}\frac{f(x)}{x^2}=$

(A) −1 (B) 0 (C) 3 (D) 2

Ans: (B)

48. Which one of the following observations is correct for the features of logarithm function to any base b > 1?

(A) The domain of the logarithm function is R, the set of real numbers.

(B) The range of the logarithm function is R^+, the set of all positive real numbers.

(C) The point (1, 0) is always on the graph of the logarithm function.

(D) The graph of the logarithm function is decreasing as we move from left to right.

Ans: (C)

49. If $P=\begin{bmatrix} 1 & \alpha & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{bmatrix}$ is the adjoint of a 3×3 matrix A and |A| = 4, then α is equal to

(A) 4 (B) 5 (C) 11 (D) 0

Ans: (C)

50. If $A=\begin{vmatrix} x & 1 \\ 1 & x \end{vmatrix}$ and $B=\begin{vmatrix} x & 1 & 1 \\ 1 & x & 1 \\ 1 & 1 & x \end{vmatrix}$, then $\frac{dB}{dx}$ is

(A) 3A (B) −3B (C) 3B + 1 (D) 1 − 3A

Ans: (A)

51. If $f(x)=xe^{x(1-x)}$ then f(x) is

(A) increasing in $\mathbb{R}$

(B) decreasing in $\mathbb{R}$

(C) decreasing in $\left[-\frac{1}{2},1\right]$

(D) increasing in $\left[-\frac{1}{2},1\right]$

Ans: (D)

52. $\int\frac{\sin x}{3+4\cos^2 x}dx=$

(A) $-\frac{1}{2\sqrt{3}}\tan^{-1}\left(\frac{2\cos x}{\sqrt{3}}\right)+C$

(B) $\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{\cos x}{3}\right)+C$

(C) $\frac{1}{2\sqrt{3}}\tan^{-1}\left(\frac{\cos x}{3}\right)+C$

(D) $-\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2\cos x}{3}\right)+C$

Ans: (A)

53. $\int_{-\pi}^{\pi}(1-x^2)\sin x \cdot \cos^2 x dx =$

(A) $\pi-\frac{\pi^2}{3}$ (B) $2\pi-\pi^3$ (C) $\pi-\frac{\pi^3}{2}$ (D) 0

Ans: (D)

54. The function $x^x; x>0$ is strictly increasing at

(A) $\forall x \in \mathbb{R}$ (B) $x<\frac{1}{e}$ (C) $x>\frac{1}{e}$ (D) $x<0$

Ans: (C)

55. The maximum volume of the right circular cone with slant height 6 unit is

(A) $4\sqrt{3}\pi$ cubic units (B) $16\sqrt{3}\pi$ cubic units

(C) $3\sqrt{3}\pi$ cubic units (D) $6\sqrt{3}\pi$ cubic units

Ans: (B)

56. The vectors $\overrightarrow{AB}=3\hat{i}+4\hat{k}$ and $\overrightarrow{AC}=5\hat{i}-2\hat{j}+4\hat{k}$ are the sides of a ΔABC. The length of the median through A is

(A) $\sqrt{18}$ (B) $\sqrt{72}$ (C) $\sqrt{33}$ (D) $\sqrt{288}$

Ans: (C)

57. The volume of the parallelopiped whose co-terminous edges are $\hat{j}+\hat{k}, \hat{i}+\hat{k}$ and $\hat{i}+\hat{j}$ is

(A) 6 cu. units (B) 2 cu. units (C) 5 cu. units (D) 3 cu. Units

Ans: (B)

58. Let $\vec{a}$ and $\vec{b}$ be two unit vectors and θ is the angle between them. Then $\vec{a}+\vec{b}$ is a unit vector if

(A) $\theta=\frac{\pi}{4}$ (B) $\theta=\frac{\pi}{3}$ (C) $\theta=\frac{2\pi}{3}$ (D) $\theta=\frac{\pi}{2}$

Ans: (C)

59. The solution of $e^{\frac{dy}{dx}}=x+1, y(0)=3$ is

(A) $y-2=x\log x-x$ (B) $y-x-3=x\log x$

(C) $y-x-3=(x+1)\log(x+1)$ (D) $y+x-3=(x+1)\log(x+1)$

Ans: (D)

60. The family of curves whose x and y intercepts of a tangent at any point are respectively double the x and y coordinates of that point is

(A) $xy=C$ (B) $x^2+y^2=C$ (C) $x^2-y^2=C$ (D) $\frac{y}{x}=C$

Ans: (A)

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